**GCSE Mathematics**

**1MA1**

**Problem-solving questions 3**

**Higher Tier: Gold**

**Time: 1 hour 30 minutes**

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Calculator not permitted in questions with ˠ

Questions with \* could be seen on Foundation Tier

**\***ˠ **1.** Alex, Benny and Clare work out the value of

 

They write their results to 3 decimal places.

Anna 1.002

Benny 10.021

Clare 100.215

Who could be correct?

You must show your working.

**(Total for question 1 is 2 marks)**

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**\*2.** The diagram shows the side, *ABCDE*, of a garage.

*ABC* is an isosceles triangle.

*ACDE* is a rectangle.

*A*

*B*

*C*

*D*

*E*

*F*

Total height

*AB* = *CB*

*BC* = 3 m

*AC* = 5 m

The ratio of the height *BF* to the height *AE* is 2:3

The height of a tractor is 2.75 m.

The width of the tractor is less than 5 metres.

 To fit into the garage, the height of the tractor must be less than the height *AE*.

 Will the tractor fit into the garage?

You must give a reason.

**(Total for question 2 is 3 marks)**

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**\*3.** Ravina is making some kebab rolls.

To make one kebab roll, she needs one naan and 2 kebabs.

Pack of 6 naans

£3.25

Pack of 8 kebabs

£5.80

Ravina is going to buy enough packs to

 have exactly twice as many kebabs as naans

 and make more than 70 kebab rolls.

Work out the least amount she can spend.

**(Total for question 3 is 4 marks)**

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**4.** The diagram shows a circle with *AB* as the diameter.

*A*

*B*

*C*

*D*

68°

Angle *BAD* = 68°

*ABC* is a straight line.

Work out angle *DBC*.

Give a reason for each stage of your working.

**(Total for question 4 is 4 marks)**

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**5.** Three groups of friends are booking tickets for a concert.

The costs of the first two groups are shown below

**Group B**

5 adults

3 children

Total cost is £43.97

**Group A**

6 adults

2 children

Total cost is £46.38

Group **C** books the same concert tickets for 3 adults and 5 children.

Work out the total cost of the tickets for group C.

**(Total for question 5 is 5 marks)**

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**\*6.** Sid wants to insure his house for £215 000

He finds two insurance companies.

**Holborn Insurance**

£585 + VAT at 20%

2.5% credit card charge

**Pearson Services**

£3.60 for every £1000

7.5% discount

Sid wants to spend the least amount of money on his house insurance.

Which company should he choose?

You must show your working.

**(Total for question 6 is 5 marks)**

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**\*7.** The diagram shows a cuboid with a cylinder cut out.

27 cm

The length of the cuboid is 27 cm.

The volume of the cuboid is 2187 cm3.

The diameter of the cylinder is the same as the length of each side of the square base.

Work out the volume of the cylinder.

Give your answer correct to 3 significant figures.

**(Total for question 7 is 4 marks)**

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**\*8.** Asha wants to buy some tickets for a concert.

An adult ticket costs £12.60

A child ticket costs of the price of an adult ticket.

Asha wants to buy 3 adult tickets and 5 child tickets.

A booking fee of 8% is added to the ticket price.

3.5 % is then added for paying by a credit card.

Asha has £94 to spend on the tickets.

Does she have enough money to buy the tickets?

You must show your working.

**(Total for question 8 is 5 marks)**

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**9.** A slimming club has 90 members.

 57 members eat fish

 41 members eat meat

 15 members do not eat fish **and** do not eat meat

A member is chosen at random.

Find the probability that this member eats fish and meat.

**(Total for question 9 is 3 marks)**

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**10.** For all values of *x,*

f(*x*) = *x*2 + 3 g(*x*) = *x* + 4

Work out the values of *x* that satisfy the equation fg(*x*) + gf(*x*) = 50

.

**(Total for question 10 is 5 marks)**

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ˠ**11.** The diagram shows a rectangle *ABCD*.

*A*

*B*

*D*

*C*

*AB* = 32*x* − 1 cm

*BC* = 9 cm

The area of the rectangle is 243 cm2.

Work out the value of *x*.

You must show your working.

**(Total for question 11 is 3 marks)**

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ˠ**12.** Triangle *ABC* is mathematically similar to triangle *DEF*.

All lengths are in centimetres.

*D*

*E*

*F*

*x*2 + 6

20

*A*

*B*

*C*

5

3*x*

*BC* is the shortest side of triangle *ABC*.

Work out the exact value of *x* in the form *a* + $\sqrt{b}$.

**(Total for question 12 is 5 marks)**

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**13.** Adrian is making some orange squash.

He makes 13 litres of orange squash correct to the nearest litre.

Each glass holds 250 m*l* of orange squash correct to the nearest 10 m*l*.

Adrian has 48 glasses.

Does he have enough orange squash to fill all 48 glasses?

You must show your working.

**(Total for question 13 is 4 marks)**

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**14.** The table shows some information about some counters in a bag.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Colour** | **red** | **green** | **blue** | **yellow** |
| **Number of counters** | 3 | *x* | 2*x* | 3*x* – 1  |

A counter is chosen at random.

The probability of choosing a red counter or a green counter is 0.22

Work out the probability of choosing a yellow counter.

**(Total for question 14 is 5 marks)**

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**15.** The diagram shows a sketch of *y = x*2 *+ bx + c*

*y*

*x*

*P*

18

(*a*, 3*a*)

The curve intersects the *y*-axis at (0,18).

The point *P* with coordinates (*a*, 3*a*) is the turning point of the curve.

Work out the value of *a*.

You must show your working.

**(5)**

**(Total for question 15 is 5 marks)**

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| **Qn** | **Answer** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| **\***ˠ**1** | Benny with working | 2 | P1 process to use approximations, e.g. A1 Benny with 10 shown |
| **\*2** | No and reason | 3 | P1 process to work out *BF*, e.g.  (= 1.6583….)P1 process to work out *AE*, e.g. “1.6583” ÷ 2 × 3 (= 2.4875)A1 No as 2.4875 is less than 2.75 |
| **\*3** | £143.40 | 4 | P1 process to find common multiples, e.g. prime decomposition of 6 and 8 or at least 5 multiples of 6 and 8P1 process to find the number of packs of naans and packs of kebabs, e.g. Packs of naan = 12 and Packs of kebabs = 9P1 process to find the total cost of the packs of naans and packs of kebabs, e.g. (12 × 3.25) + (2 × 9 × 5.80)A1 £143.40 |
|  |  |  | ALTERNATIVE MARK SCHEME |
|  | £143.40 | 4 | P1 process to work out the number of packs of naans, e.g. 70 ÷ 6 (= 11.666 = 12)P1 process to find the number of packs of kebabs, e.g. 2 × 70 ÷ 8 (= 17.5 = 18)P1 process to find the total cost of the packs of naans and packs of kebabs, e.g. (12 × 3.25) + (18 × 5.80)A1 £143.40 |
| **4** | 158° and correct reasons | 4 | B1 *ADB* = 90°C1 angles in a semicircle are 90°M1 180 – 90° – 68° or 180 – 22°A1 *DBC* = 158° with reasons e.g. angles in a triangle add up to 180o, angles in a straight line add up to 180o |
|  |  |  | ALTERNATIVE MARK SCHEME |
|  | 158° and correct reasons | 4 | B1 *ADB* = 90°C1 angles in a semicircle are 90°M1 90 + 68° A1 *DBC* = 158° with reasone.g. the exterior angle of a triangle is equal to the sum of the interior opposite angles |
| **5**  | £39.15 | 5 | P1 process to set up two equations, e.g. 6*a* + 2*c* = 46.38 and 5*a* + 3*c* = 43.97 P1 process to eliminate a variable, e.g. 18*a*  –10*a* = 139.14 – 87.94 or 18*c*  –10*c* = 263.82 – 231.90 P1 process to find the second variable, e.g. 6(“6.40”) + 2*c* = 46.38 or 5(“6.40”) + 3*c* = 43.97 or 6*a*+ 2(“3.99”) = 46.38 or 5*a*+ 3(“3.99”) = 43.97 P1 process to find the cost of group C, e.g. 3 × 6.40 + 5 × 3.99 (= 39.15) A1 for £39.15  |
| **\*6** | Pearsonwithworking  | 5 | P1 process to work out the cost of insurance for Pearson services without the discount, e.g. 215 × 3.60 (= 774)P1 process to work out the cost of insurance for Pearson services with the discount, e.g. 0.925 × “774” (= 715.95)P1 process to work out the cost of insurance for Holborn Insurance with the VAT, e.g. 585 × 1.2 (= 702)P1 process to work out the cost of insurance for Holborn Insurance including VAT with the credit charge, e.g. 1.025 × “702” (= 719.55)A1 for Pearson with 715.95 and 719.55  |
| **\*7** | 1720 | 4 | P1 process to find the cross sectional area of the cuboid, e.g. 2187 ÷ 27 (= 81)P1 process to find the radius of the cylinder, e.g.  (= 9) or ÷2 (= 4.5)P1 process to find the volume of the cylinder, e.g. *π* × “4.5”2 × 27A1 1720 |
| **\*8** | No with£95.07 | 5 | P1 process to find the price of a child’s ticket, e.g. 0.75 × 12.60 (= 9.45)P1 process to find the price of the total number of tickets, e.g. (3 × 12.60 + 5 × “9.45”) (= 85.05) P1 process to find the cost of the tickets with the booking fee, e.g. 1.08 × “85.05” (= 91.854)P1 process to find the cost of the tickets with the credit card, e.g. 1.035 × “91.854” (= 95.068...) or 1.035 × “91.85” or 1.035 × “91.86”A1 No with 95.07  |
| **9** |  |  3 | P1 process to find the number of members who eat fish and eat or both, e.g. 90 – 15 (= 75)P1 process to find the number of members who eat fish and eat meat, e.g. “75” = 57 + 41 – *x* or *x* = 98 – “75” (=23)A1 for   |
|  |  |  | ALTERNATIVE MARK SCHEME |
|  |  |  3 | P1 process to draw a Venn diagram with two overlapping circles and correctly labelled, e.g. FishMeat15P1 process to find the intersection of the two overlapping circles, e.g. (57 + 41 + 15) – 90 (= 23)A1 for  |
| **10** | *x* = 2 *x* = −6 | 5 | P1 process to work out fg(*x*) or gf(*x*), e.g. (*x* + 4)2 + 3 or *x*2 + 3 + 4A1 *x*2 + 8*x* + 19A1 *x*2 + 7 P1 process to solve the equation  “' *x*2 + 8*x* + 19” + “ *x*2 + 7” = 50 e.g. 2*x*2 + 8*x* − 24 = 0 = 2(*x* + 6)(*x* − 2) = 0 A1 for *x* = 2 and *x* = −6 |
| ˠ**11** | 2 | 3 | P1 process to set up an equation of the area of the rectangle, e.g. 32*x* − 1 × 9 = 243P1 process to solve the equation of the area of the rectangle by eliminating base 3, e.g. 2*x* – 1 = 3A1 2 |
| ˠ**12** | 6 + | 5 | P1 process to set up a quadratic equation for similar triangles, e.g.  or *x*2 + 6 = 4(3*x*) P1 process to simplify the quadratic equation into the form a*x*2 + b*x* + c = 0, e.g. *x*2 – 12*x* + 6 = 0P1 process to solve the quadratic equation, e.g.  or P1 process to simplify the expression, e.g. A1 6 + |
| **13** | Yes and reason | 4 | P1 process to find a lower bound or an upper bound for 13 litres, e.g. 13 ± 0.5 (= 12.5 or 13.5) or 13000 ± 500 (= 12500 or 13500)P1 process to find a lower bound or an upper bound for 250 m*l*, e.g. 250 ± 5 (= 245 or 255) or 0.250 ± 0.005 (= 0.245 or 0.255)P1 process to find the number of glasses or the total volume of orange squash, e.g. 12.5 ÷ 0.255 or 12500 ÷ 255 (= 49.0196….) or 48 × 0.255 (= 12.24) or 48 × 255 (= 12240)A1 Yes as 48 < 49 or 12.24 < 12.5 oe |
| **14** |  | 5 | P1 process to find the probability of a red or green counter OR blue or yellow counter, e.g.  or P1 process to find the equation of a red or green counter OR blue and yellow counter, e.g. = 0.22 or = 0.78P1 process to solve the equation, e.g. (3 + *x*) = 0.22(3 + *x* + 2*x* + 3*x* – 1) or (2*x* + 3*x* – 1) = 0.78(3 + *x* + 2*x* + 3*x* – 1) or (3 + *x*) = 0.22(2 +6*x*) or (5*x* – 1) = 0.78(2 +6*x*) or *x* = 8P1 process to find the probability of a yellow counter, e.g. (3 × “8” – 1) ÷ (3 + “8” + 2 × “8” + 3 × “8” – 1)A1  oe |
| **15** | 3 | 5 | P1 process to set up an equation which includes the value of *c*, e.g. *x*2 + *bx* + 18P1 process to set up an equation for point *P*, e.g. (*x* – *a*)2 + 3*a*P1 Process to set up an equation to find the value of *a*,e.g. (*x* – *a*)2 + 3*a* = *x*2 + b*x* + 18 or *x*2 − 2*ax* + *a*2 + 3*a* = *x*2 + b*x* + 18 or *a*2 + 3*a* = 18P1 process to solve a quadratic equation of the form a*x*2 + b*x* + c = 0, e.g. (*a* – 3)(*a* + 6) = 0A1 3 (can ignore *a* = −6 for A1) |